



## Investigation on the viscoelasticity of optical glass in ultraprecision lens molding process

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### ABSTRACT

Glass molding press is an efficient manufacturing technology for ultraprecision optical elements with complex shapes. In glass molding, viscoelastic property of glass is an essential aspect that determines the glass deformation behavior around the molding temperature. In this paper, viscoelasticity of glass has been measured experimentally by uniaxially compressing cylindrical glass preforms above the glass transition temperature using an ultraprecision glass molding machine. The elastic modulus and viscosity of glass were obtained by curve fitting techniques using the Burgers model and the Maxwell model for creep and stress relaxation, respectively. Based on the thermo mechanical and viscoelastic parameters obtained from experiments, finite element model simulations of the glass molding process were performed, which can be used to visualize the stress/strain distribution and to predict the residual stress in glass.

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### 1. Introduction

Glass lenses, including spherical and aspherical lenses, have been widely used in optical, optoelectrical and optomechanical systems. Aspherical lenses are able to deliver much better optical performance and image quality than traditional spherical lens by eliminating spherical aberration, increasing light permeability, and reducing size and weight of optical assemblies. Traditional methods to produce aspherical lenses are material removal processes, such as diamond turning, grinding and polishing technologies (Nicholas and Boon, 1981; Johnson and Michael, 2005). These manufacturing techniques can be used to fabricate high-accuracy lenses with very smooth surfaces. However, they are uneconomic and unpractical for mass production of aspherical lenses needed in household products such as digital cameras, CD/DVD players and recorders and mobile phones.

In this background, glass molding press (GMP) has recently emerged as a promising way to mass-produce aspherical glass lenses (Jain and Yi, 2005; Katsuki, 2006; Masuda et al., 2007). In GMP, a glass lens is produced by compressing glass preforms at a high temperature and replicating the shapes of the mold cores to the lens surface without need for further machining process. GMP has remarkably higher production efficiency than the conventional material removal processes. However, because the heat transfer phenomenon and high-temperature thermal and mechan-

ical characteristics of glass have not been fully clarified, there are still uncertainties and bottlenecks in the GMP technology (Yan et al., 2009). As a result, shape accuracy of the molded lens changes significantly with molding conditions and optical properties of the lens vary with residual stress distribution within the lens. What's more, the molds' service lives are very limited and sensitive to the molding temperature and pressing load, and so on (Aono et al., 2000; Hall et al., 2005; Kim et al., 2007).

Finite element method (FEM) simulation of the glass molding process is an effective way to optimize the molding conditions instead of the experimental trial-and-error method. A few previously reported FEM simulations of the GMP process are based on the elastic-plastic models of glass (Stokes, 2000). However, because GMP is usually done at a temperature between the transition point ( $T_g$ ) and the softening point (SP) of glass, the glass material shows significant viscoelasticity in deformation. It is the viscoelasticity that gives rise to creeping phenomenon in the pressing and stress relaxation in the annealing stages. At a much higher temperature near SP, glass behaves like a viscous liquid (Chang et al., 2007; Yan et al., in press). Therefore, in order to determine the molding conditions to improve the quality of the molded lens, clarification of the viscoelasticity and viscosity of glass at a high temperature is very important. In a few recent studies on glass lens molding processes, the viscoelastic properties of glass have been taken into account (Jain and Yi, 2005; Na et al., 2007), but the parameters in their viscoelastic models were lack of experimental confirmation.

In the present work, we investigated the viscoelasticity of glass by carrying out high-temperature GMP experiments and analyzing the creeping and stress relaxation behaviors of glass. Uniaxial

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compression using a pair of flat molds was used to perform creeping and stress relaxation, and the temperature-dependent elastic modulus  $E$  and viscosity  $\eta$  of the glass were measured. Then, numerical models were built based on the experimental results, with which the stress/strain changes of glass during pressing and annealing were tracked. By integrating numerical simulation and experimental validation, the viscoelastic deformation of glass can be precisely modeled. The FEM simulation based on this viscoelastic model can be used to precisely predict the optimal molding conditions, such as pressing load and pressing rate. It is expected that the outcome of this work contributes to improvement of the form accuracy and the optical performance of the molded glass components, such as aspherical lens, Fresnel lenses, diffractive optical elements (DOEs), and other complex surface microstructures.

## 2. Theory

### 2.1. Theory of viscoelasticity

In a typical glass lens molding cycle, a piece of glass preform is heated to the molding temperature several tens of degrees centigrade above its glass transition temperature, then the preform is compressed between the pair of molds to replicate the shapes of cores. After that, the stress is relaxed by holding the pressing load for a short time at a slow cooling rate. Finally, the formed glass lens is cooled to ambient temperature rapidly and released from the molds. Hence the whole molding cycle can be divided into four stages: heating, pressing, annealing and cooling, as schematically shown in Fig. 1.

At the molding temperature, glass shows viscoelasticity in deformation. Viscoelasticity is a combination of viscous and elastic characteristics when undergoing deformation. Viscoelastic materials exhibit time-dependent strain under constant stress, and decreasing stress under constant strain, which are termed as “creep” and “stress relaxation”, respectively. As glass is an isotropic material, the viscoelasticity of glass, including creep and stress relaxation, can be described by an integral form of the stress–strain constitutive equation as follows (Findley et al., 1976):

$$\sigma + p_1 \dot{\sigma} + p_2 \ddot{\sigma} = q_0 \varepsilon + q_1 \dot{\varepsilon} + q_2 \ddot{\varepsilon} \quad (1)$$

where  $\sigma$ ,  $\dot{\sigma}$ ,  $\ddot{\sigma}$  are stress, first derivative of stress and second derivative of stress, respectively, of glass during uniaxial compression.

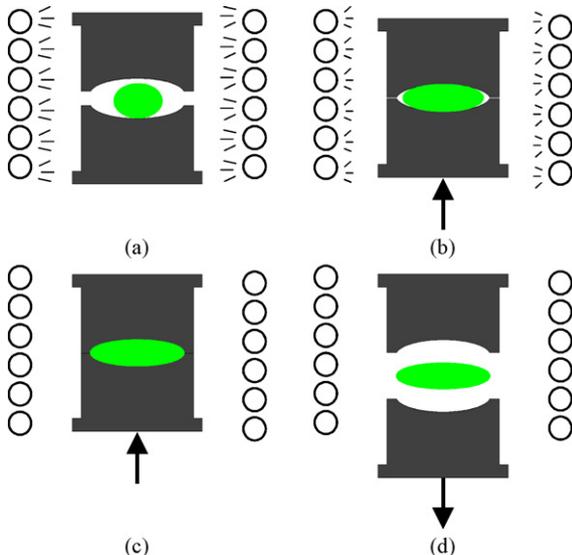


Fig. 1. Schematic diagram of four stages of a GMP process cycle: (a) heating, (b) pressing, (c) annealing, and (d) cooling.

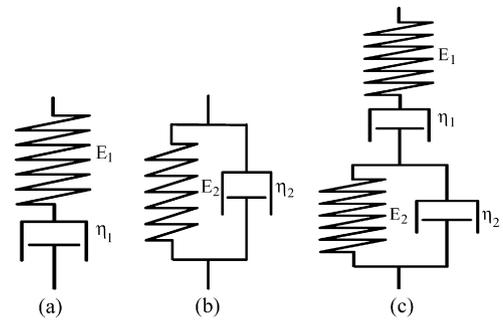


Fig. 2. Viscoelastic models for glass in transition region: (a) Maxwell model, (b) Kelvin model, and (c) Burgers model.

$\varepsilon$ ,  $\dot{\varepsilon}$ ,  $\ddot{\varepsilon}$  are strain, first derivative of strain and second derivative of strain, respectively.  $p_1$ ,  $p_2$ ,  $q_0$ ,  $q_1$ ,  $q_2$  are constants related to the elastic modulus and viscosity of a particular material, which should be defined by experiments. Both the creep behavior and the stress relaxation can be derived from this differential equation. In order to properly define request parameters and solve the second order differential equation, simplification of the equation is necessary. Generally, mechanical models involving springs and dashpots are devised to physically interpret the creep and the stress relaxation of viscoelastic deformation, which can obtain an explicit formulation instead of solving the differential equation.

Among various mechanical models, Maxwell model, Kelvin model and Burgers model are three typical models which combine a series of elements with springs and dashpots (Pipkin, 1972). The Maxwell model is a two-element model consisting of a linear spring element and a linear viscous dashpot element connected in series as shown in Fig. 2(a). The Kelvin model is shown in Fig. 2(b), where a spring element and dashpot element are connected in parallel. The Burgers model is shown in Fig. 2(c), where a Maxwell and a Kelvin model are connected in series.

### 2.2. Creep

When applying a specified stress on glass at the molding temperature, strain  $\varepsilon(t)$  goes on increasing with time  $t$ , which is called creep. In creep, the deformation of glass is a superposition of instantaneous elastic deformation, delayed elastic deformation and viscous flow, which can be described by viscoelastic mechanical models. When applying a constant stress  $\sigma_0$  at  $t = 0$ , the strain–time relationship can be obtained by Eq. (2) according to the Maxwell model. For Kelvin model, the stress–strain relationship is defined by Eqs. (3a) and (3b). The creeping behavior of the Burger model under constant stress  $\sigma_0$  can be obtained by Eqs. (4a) and (4b):

$$\varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0 t}{\eta_1} \quad (2)$$

$$\varepsilon(t) = \frac{\sigma_0}{E_2} (1 - e^{-t/\tau_{kd2}}) \quad (3a)$$

$$\tau_{kd2} = \frac{\eta_2}{E_2} \quad (3b)$$

$$\varepsilon(t) = \frac{\sigma_0}{E_1} + \frac{\sigma_0}{E_2} (1 - e^{-t/\tau_{bd2}}) + \frac{\sigma_0 t}{\eta_1} \quad (4a)$$

$$\tau_{bd2} = \frac{\eta_2}{E_2} \quad (4b)$$

where  $E_1$ ,  $E_2$ ,  $\eta_1$ ,  $\eta_2$  are elastic modulus and viscosities of the viscoelastic material. Usually in Eqs. (3a) and (4a),  $\tau_{kd2}$  and  $\tau_{bd2}$  are defined as the term “retardation time” to describe the creep rate in the viscoelastic deformation, and given by Eqs. (3b) and (4b), respectively. Additionally, in the pressing stage, as the glass preform is compressed at a uniform temperature distribution (Yan et

al., 2009), the strain analysis in creep can be treated as an isothermal problem without considering heat transfer.

### 2.3. Stress relaxation

If we suddenly impose a strain onto a body and with the strain held constant, then the stress  $\sigma(t)$  varies with time  $t$ . This behavior is called stress relaxation. Stress relaxation in high-temperature glass has been studied extensively by previous researchers (Christensen, 1982). Viscoelastic stress relaxation can also be illustrated with the help of Maxwell model, Kelvin model and Burgers model. If a step strain  $\varepsilon(t_0)$  is imposed at time  $t_0$  (the instantaneous stress reaches  $\sigma(t_0)$ ) and held constant, then  $\varepsilon(t) = \varepsilon(t_0)h(t - t_0)$ , where  $h(t)$  is a unit step function. The response functions to describe the stress relaxation can be derived from Eq. (1) according to the three mechanical models. Stress relaxation equations based on the Maxwell model, the Kelvin model and the Burgers model are listed in Eqs. (5a), (5b), (6), (7a), (7b) and (7c), respectively:

$$\sigma(t) = \sigma(t_0)e^{-(t-t_0)/\tau_{ms1}} \quad (5a)$$

$$\tau_{ms1} = \frac{\eta_1}{E_1} \quad (5b)$$

$$\sigma(t) = \sigma(t_0) \left( 1 + \frac{\eta_2}{E_2} \delta(t - t_0) \right) \quad (6)$$

$$\sigma(t) = \frac{\sigma(t_0)}{\sqrt{(E_1\eta_1 + E_1\eta_2 + E_2\eta_1)^2 - 4E_1E_2\eta_1\eta_2}} \times \left[ \left( E_2\eta_1 - \frac{\eta_1\eta_2}{\tau_{bs1}} \right) e^{-(t-t_0)/\tau_{bs1}} - \left( E_2\eta_1 - \frac{\eta_1\eta_2}{\tau_{bs2}} \right) e^{-(t-t_0)/\tau_{bs2}} \right] \quad (7a)$$

$$\tau_{bs1} = \frac{2\eta_1\eta_2}{(E_1\eta_1 + E_1\eta_2 + E_2\eta_1) - \sqrt{(E_1\eta_1 + E_1\eta_2 + E_2\eta_1)^2 - 4E_1E_2\eta_1\eta_2}} \quad (7b)$$

$$\tau_{bs2} = \frac{2\eta_1\eta_2}{(E_1\eta_1 + E_1\eta_2 + E_2\eta_1) + \sqrt{(E_1\eta_1 + E_1\eta_2 + E_2\eta_1)^2 - 4E_1E_2\eta_1\eta_2}} \quad (7c)$$

where  $\tau_{ms1}$ ,  $\tau_{bs1}$  and  $\tau_{bs2}$ , defined by Eqs. (5a), (7b) and (7c), are termed relaxation times and are constants at a specified temperature. As the temperature of glass keeps decreasing in the annealing stage of a molding process, strictly speaking, the stress analysis in stress relaxation should be treated as a nonisothermal problem with considering the heat transfer between glass and molds (Yan et al., 2009). In this study, the stress relaxation behavior is measured at a series of constant temperatures for approximation.

## 3. Experiments and simulations

### 3.1. Glass molding tests

The GMP experiments were carried out using an ultraprecision glass molding machine GMP211 (Toshiba Machine Corp., Shizuoka, Japan). In order to simplify calculation procedure of the parameters of the mechanical models, a pair of flat molds made of tungsten carbide (WC) with surface coatings was used to press cylindrical glass preforms. Nitrogen gas is used to purge the air in the molding chamber to prevent the molds from oxidation at high temperatures.

**Table 1**  
Thermo-mechanical properties of glass L-BAL42.

Property	Value
Specific gravity $d$	3.05
Modulus of elasticity $E$ ( $\times 10^8$ N/m <sup>2</sup> )	891
Modulus of rigidity $G$ ( $\times 10^8$ N/m <sup>2</sup> )	357
Poisson's ratio $\nu$	0.247
Transition temperature $T_g$ ( $^{\circ}$ C)	506
Yielding temperature $A_t$ ( $^{\circ}$ C)	538
Softening point $SP$ ( $^{\circ}$ C)	607

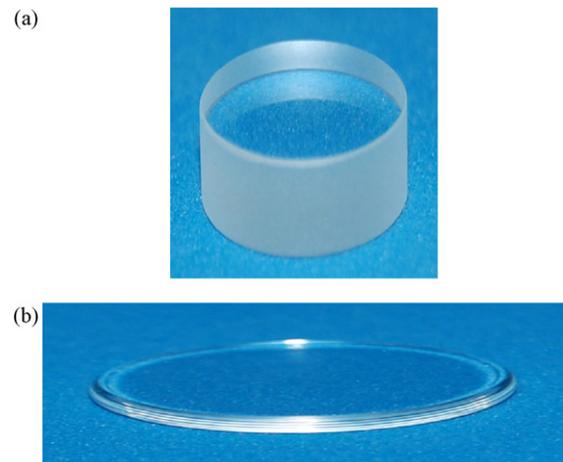
The molding chamber is covered by a transparent silica glass tube, which can let the infrared rays in and separate the nitrogen gas from the air outside. Temperatures of the upper and lower molds are monitored by two thermocouples beneath their surfaces with a measurement accuracy of  $\pm 1$   $^{\circ}$ C. The upper mold is remained stationary, while the lower mold is driven upward and downward by an AC servomotor with a resolution of 0.1  $\mu$ m. A load cell is placed beneath the lower axis as a feedback of the pressing load with a resolution of 0.98 N. The four-stage molding process was realized by a G-code computer program. During experiments, the temperatures of the upper and lower molds, the position of the lower mold, the pressing load applied to the molds were recorded with respect to time, respectively.

The glass material used in the present work is L-BAL42 (Ohara Inc., Kanagawa, Japan), 4.7 mm long and 12 mm in diameter. Thermo-mechanical properties of the glass material are listed in Table 1. Photographs of a cylindrical glass preform and a molded flat piece are shown in Fig. 3(a) and (b), respectively.

As discussed in Section 2, all the equations were derived at a particular temperature. In the present study, to simplify calculation, both the creep and the stress relaxation were carried out at the same temperature as isothermal processes. Fig. 4 schematically shows the changes of pressing load and mold displacement during creep and stress relaxation in time sequence. The creep was conducted for 100 s under the same pressing load of 1000 N (stress  $\sigma_0 = 8.846$  MPa) at a series of temperatures from 560 to 590  $^{\circ}$ C in the torque control mode. Then a step displacement was imposed to the lower mold and the position was held constant for a period of 60 s to conduct stress relaxation.

### 3.2. Viscoelastic deformation

The changes of strain with time during creep at different temperatures is shown in Fig. 5. It can be seen that the delayed elastic and



**Fig. 3.** Photographs of (a) a cylindrical glass preform before experiment and (b) a molded flat piece.

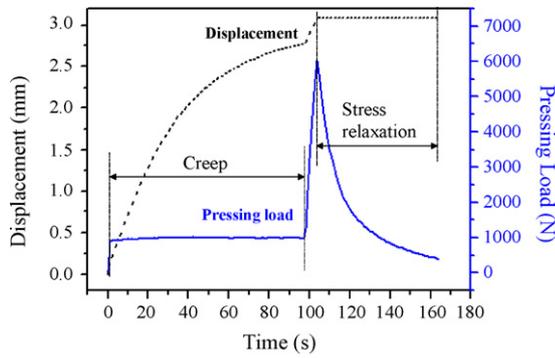


Fig. 4. Plots of mold displacement and pressing load in time sequence during creep and stress relaxation of glass.

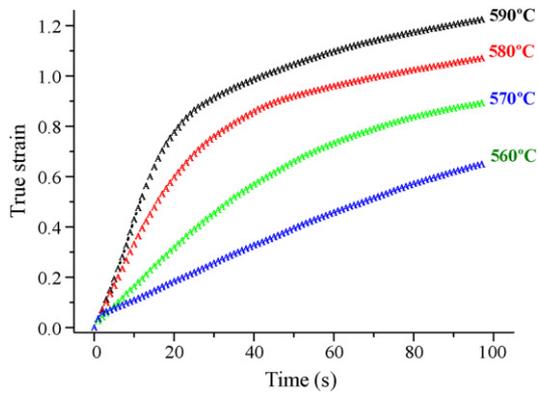


Fig. 5. Changes of true strain with time during creep at different temperatures.

viscous flow components vary with time, and contribute an appreciable amount to the total deformation. The delayed elastic and the viscous flow are significant at higher temperatures, and show rapid increases in the true strain within the first 20 s.

Next, the least square nonlinear curve fitting technique was used to fit the experimental results based on the three viscoelastic models. In the Maxwell model, elastic modulus  $E_1$  and viscosity  $\eta_1$  increase when the temperature gets lower, which can be used to explain the phenomena that smaller elastic deformation and less viscous flow occur at a lower temperature. In the Kelvin model, elastic element  $E_2$  decreases, while viscosity  $\eta_2$  increases as temperature decreases, which means the retardation time  $\tau_{kd2}$  of the elastic deformation is longer at a lower temperature. The Burgers model combines the characteristics of both the Maxwell model and the Kelvin model.

Fig. 6 is plots of curve fitting results by using the three viscoelastic models. We can see that the Maxwell model is not able to fit the

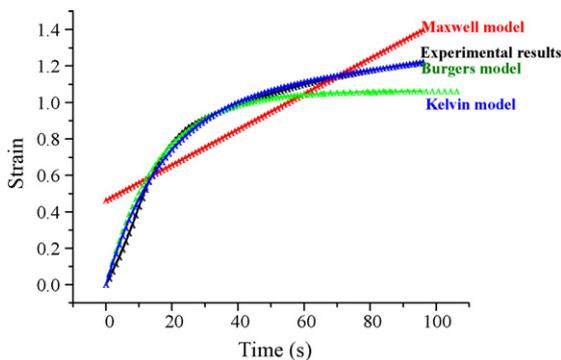


Fig. 6. Curve fitting results of creep at 590 °C by using three viscoelastic models.

Table 2  
Parameters of the Burgers model for creep.

Parameters	$E_1$	$\eta_1$	$E_2$	$\eta_2$	$\tau_{bd2}$
590 °C	44592.0	4105.2	8.7	150.1	17.3
580 °C	51872.8	15094.7	8.6	206.8	24.0
570 °C	64969.5	28389.1	8.4	452.2	53.8
560 °C	72333.3	46517.9	8.0	910.1	113.8

creep curve, and the Kelvin model can fit the creep curve with a deviation in later stage. However, the Burgers model coincides with the experiment results perfectly. Therefore, the Burgers model was adopted as the best fit in this paper to build the numerical model of creep. From the curve fitting results, the parameters of the Burgers model at different temperatures are listed in Table 2. It can be seen that the lower the temperature is, the longer the retardation time  $\tau_{bd2}$  is.

### 3.3. Annealing

Annealing is to a stage for releasing the internal stress by shortly holding a specified pressing load after the pressing stage. Since the residual stress in the glass lens is decided by the annealing stage, a deep understanding of the stress change during annealing is important. Theoretically, the strain of the molded glass does not change in annealing when the molds have been closed. However, this is not the case for an actual molding process. As shown in Fig. 7(a), the pressing load  $F$  is the sum of the load imposed on the in-process glass lens  $F_1$  and the load imposed on the molds  $F_2$ . As a constant  $F$  is maintained in the torque control mode during annealing,  $F_2$  increases with time due to that  $F_1$  is decreasing by the stress relaxation of glass. The increase in  $F_2$  will lead to a small change in elastic distortion of the molds, which cannot be neglected in an ultraprecision molding process.

In this study, an experiment was specially designed to approximate the change of  $F_1$ . Despite that in actual GMP processes, creep and stress relaxation are both performed in the torque control mode, in the present experiments, both the torque control mode and the position control mode were used. During creeping, the press load is controlled in the torque control mode, where the displacement goes on increasing to a specified value just before the molds are closed. Then the torque control mode is shifted to the position control mode, and the position of lower mold is held. This situation is very similar to that of annealing when molds have been closed. In this case, the change of pressing load  $F$  is equal to the load imposed on the in-process glass lens  $F_1$ , as shown in Fig. 7(b), which can be monitored by the load cell. Annealing is usually conducted at a changing temperature from the molding temperature above  $T_g$  to an annealing-end temperature below  $T_g$ . In this research, the

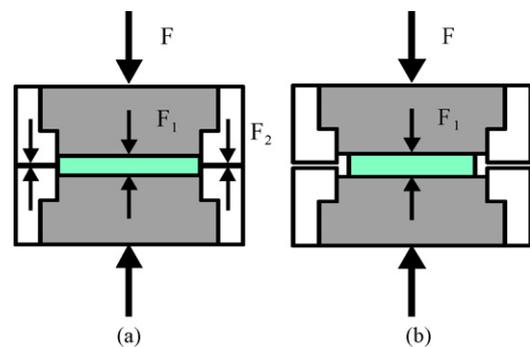
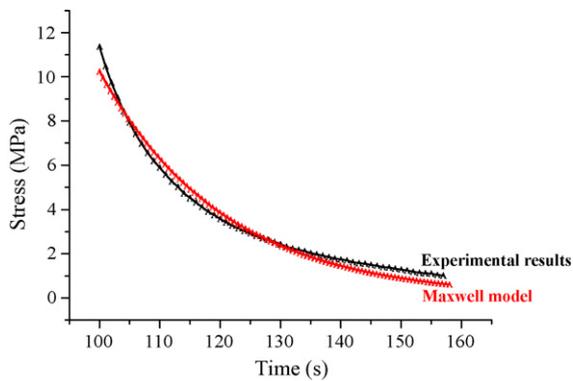


Fig. 7. Schematic diagram of force balances during annealing in (a) an actual GMP process based on torque control mode and (b) a special experiment based on the position control mode.



**Fig. 8.** Plots of stress changes with time during stress relaxation at 590 °C, shown together with a curve fitted by the Maxwell model.

**Table 3**  
Parameters of the Maxwell model for stress relaxation.

Parameters	$E_1$	$\eta_1$	$\tau_{ms1}$
590 °C	7.3	143.8	20.5
580 °C	9.6	168.1	17.6
570 °C	15.1	250.8	16.6
560 °C	19.3	259.4	13.4

stress relaxation was tested at a series of constant temperatures for approximation.

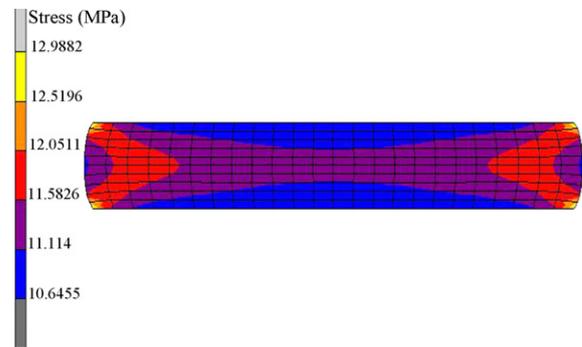
Among the three mechanical models mentioned in Section 2, the Kelvin model is unable to describe the time-dependent change of stress. The Burgers model would be the most accurate one to describe stress relaxation, but practically it is very difficult to identify the four parameters in the model. In this work, we used the Maxwell model to fit the stress relaxation. Fig. 8 shows changes of stress with time in the annealing stage at a temperature of 590 °C, together with the curve fitted by the Maxwell model. From the figure, we can find that the experimental curve and the fitted curve are generally consistent, and that the stress relaxes to almost zero after holding for 60 s.

The parameters of Maxwell model in different temperatures are listed in Table 3. The elastic modulus and viscosity increase as the temperature gets lower. The lower the temperature is, the shorter the stress relaxation time  $\tau_{ms1}$  is. That is to say, a less holding-time is needed in annealing at a higher temperature. In glass, the viscosity is a steep function of temperature. Therefore, as the temperature rises beyond the transition point, the viscosity of glass falls sharply during heating, resulting in a decrease of retardation time and a rapid increase of the creep rate which sharply reduces the pressing load. In reverse, the viscosity of glass rises sharply when the temperature goes down during annealing, resulting in very low-relaxation time and decrease of the stress relaxation rate.

### 3.4. FEM simulations

The FEM simulations were performed by a commercial nonlinear FEM program MSC.Marc, which can simulate static, dynamic and coupled physics problems for a wide range of design and manufacturing application. The viscoelastic strain and stress analyses were conducted by this code pack. The cylindrical glass compressing process was simulated by using a central plate two-dimensional FEM model, meshed with quadrilateral elements. The thermal–mechanical properties of the glass material were set as a viscoelastic model according to the constitutive equations and experimental results in Sections 2 and 3.

Fig. 9 shows a simulation result of equivalent stress (von Mises stress) distribution in the deformed glass when the displacement



**Fig. 9.** FEM simulated result of von Mises stress distribution during creep at 590 °C.

of creep reaches 2.5 mm at 590 °C (pressing rate = 0.05 mm/s). The glass material flows in a large strain at the outer regions where the stress is much higher than that in the center. When the molds are closed, the stress reaches its maximum value. Subsequently, annealing will be performed and the stress is finally relaxed to zero. As stress relaxation will be conducted from the molding temperature to the annealing-end temperature, the calculation of the least time to relieve the internal stress to zero is a very complex issue. The stress relaxation process of a molded lens is to be further investigated.

## 4. Conclusions

Viscoelastic properties of glass in ultraprecision lens molding process were studied by cylindrical compressing. Creep and stress relaxation of glass at high temperatures have been studied through theoretical analysis, experiments and FEM simulations. The main findings of the present work are summarized as follows:

- (1) In glass molding press, viscoelastic material flow is a major component of deformation in the pressing stage. The viscoelasticity of glass above glass transition temperature can be obtained by tracking strain and stress changes during creep and stress relaxation.
- (2) Uniaxial pressing of cylindrical glass preforms between a pair of flat molds in a GMP machine is an effective way to carry out creep and stress relaxation experiments.
- (3) The Burgers model is suitable for describing the creep behavior, the elastic and viscous parameters of the model can be obtained by curve fitting. The retardation time gets longer at lower temperatures during creep.
- (4) The Maxwell model can be used for fitting the stress relaxation of glass. As the temperature gets lower during annealing, the stress relaxation time gets shorter.
- (5) Material flow, strain change during creep, and stress change during stress relaxation can be visualized by FEM simulation based on the Burgers model and Maxwell model, which is helpful in optimization of molding conditions.

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